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② **LEVEL** II

⑥ IMAGE EVALUATION USING MODULATION TRANSFER
FUNCTIONS FOR CASSEGRAINIAN - TYPE
TELESCOPIC SYSTEMS
⑨ Technical Report

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Introduction

When an optical system is being used, the quality of its performance is always the main concern. The existence of aberration and diffraction in image formation by mirrors and lenses were well appreciated, even back in the 17th century. Rayleigh's $\lambda/4$ limit and "the Strehl ratio" are among the most accepted as the creterion for permissible wavefront errors. However the Strehl ratio, like the Rayleigh creterion, is only useful for highly corrected system. In recent years, the application of more sophisticated detectors has magnified the problem of determining the quality of an optical system, or system-and sensor combination. The introduction of the concept of transfer function has played an important part in improving the technique of assessment.

The transfer function is valuable and powerful because it has many advantages;

i) The validity of using an optical transfer function rests only on the two simple postulates, superposition and shift variance. The former is satisfied for the intensity in the image of an incoherent object, and the latter merely requires that the form of the image of a point source, that is the point spread function, be invariant over a small region of the image plane.

ii) The applicability of the transfer function to image formation is independent of any specific theory of light, of the spectral composition of the light, of the shape of the aperture of the optical system, and also of the type & magnitude of aberration, provided they are not too large to satisfy the requirement of the shift invariance.

iii) The optical transfer function may be calculated directly from the design data of any system, and it may also be measured for the system under testing. It has thus made possible the prediction and checking of the expected image quality.

iv) One of the greatest advantages to be gained by using optical
transfer function is that of cascading system elements. This
cascading property permits the lens transfer function to be
combined with that of the detectors, be it a photographic film,
a image tube or even our eyes. In general, we can simplify the
optical system by considering the cascading process. The system
transfer function may be written as

$$T(R) = \prod_1^n T_m(R) \tag{1}$$

where   R: spatial frequency
         m: the mth component in the optical system

For our specific interest[*], evaluating the optical system per-
formance characteristic of airbone designators, the optical sys-
tem may be considered as Figure 1. Because of the linear rela-
tion between each component, the system transfer function is
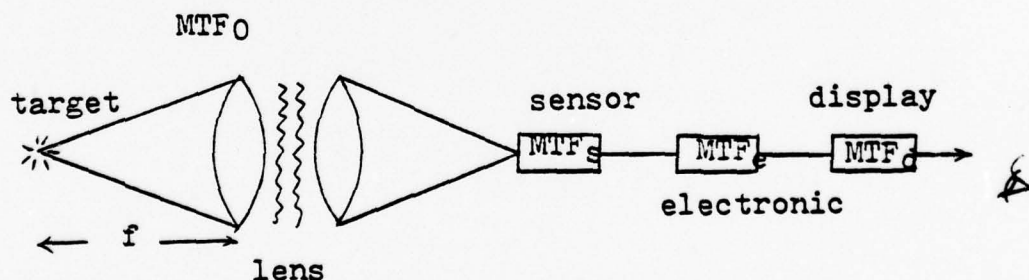simply the product of the transfer function of each component.

Figure 1   Components of a typical optical system

This study consists of reviewing the theory, calculation and measurement of the transfer function, and is discussed into the following sequences;

      I. Concepts and Definations
     II. Calculation of Transfer Function
    III. Measuring Instruments and Techniques

The design concept of an optical measuring system is based on the technical evaluation of the performance of a cassegrain telescope.   The effects on modulation transfer function, due to wavefront error, image motion and central obstruction when cassegrain telescopes are used, are examined.

The choice of suitable criteria and method for testing is discussed.   Results and data analysis are given as how to perform and judge the performance of an optical system based on MTF test from a simple laboratory alignment.

I. Basic Concepts and Definations

The performance specification of an optical system is primarily
intended to describe its ability to produce an image which will be
a true representation of the object being viewed.   The quality of
the image formed by an optical system will be mainly determined by
the following three factors;
    a. aberration in the optical system
    b. wave nature of light
    c. inaccuracy incurred due to manufacturing processes
When a system design is such that the aberration can be neglected
and the errors of manufacturing are so small that they nearly have
no effect on the performance, the quality of the image formed by
the system is said to be diffraction limited. Thus the image of a
bright point formed by an optical system is not a geometrical point
in the image plane, but a light distribution consisting of a small
bright core surrounded by a more or less extensive halo. Consider
a lens system which forms images of an incoherently illuminated
narrow slit. In the ideal case, the image would be an exact replica
of the object, as demonstrated in Figures 2a and 2b. This can never
be realized because of diffraction, the best that can be achieved is
the familiar distribution of intensity in Figure 2c. If now there
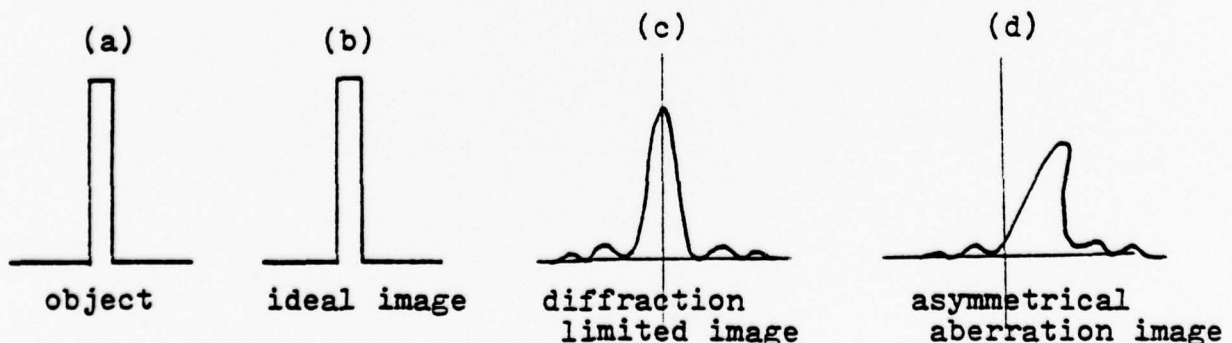is aberration the image is much more complex, as illustrated in Fig.
2d.

(a)	(b)	(c)	(d)
object	ideal image	diffraction limited image	asymmetrical aberration image

Figure 2   Images

The intensity distribution in the image of a narrow incoherently illuminated slit is know as the line spread function of the optical system forming the image. If the slit is replaced by a pinhole, the corresponding function is referred to as the point spread function. A sinusoidal target is considered the best, because the image of a sinusoidal target always has intensity distribution that is sinusoidal and similar to the form of the target.

The effect of diffraction is to reduce the amplitude of the image intensity distribution.   If, in addition, a aberration is presented the amplitude would be further reduced.   The amplitude reduction due to aberration is accompanied by a phase change.

Let us consider a series of sinusoidal targets of varying spatial frequency but of constant amplitude as subsquent test objects. If N is the distance between successive peaks of the target (in millimeter), the spatial frequency of the target is defined as

$$R = 1/N \quad \text{cycles/mm} \tag{3}$$

The images of the above targets will be of reduced amplitudes, and the corresponding normalized amplitudes, the modulation, can be then calculated for each spatial frequency. The variation of modulation with spatial frequency defined its Modulation Transfer Function, i.e., MTF.   Figures 3 & 4 give the concept of modulation and typical MTF curve respectively. If the light distribution in the object and the corresponding image is analyzed by Foruier Transform, the MTF may be defined as following,

$$\text{MTF} = \frac{\text{Fourier Transform of Image Intensity Distribution}}{\text{Fourier Transform of Object Intensity Distribution}}$$

In the case of a lens possessing aberration, the effect of the lens system is to cause a reduction in modulation, accompanied by a phase change.   This phase change is spatial frequency dependent, and it contributes to what is known as the Phase Transfer Function (PTF). Measurement of MTF and PTF combine to give the Optical Transfer Function (OTF).

$$\text{modulation} = \frac{B_{max} - B_{min}}{B_{max} + B_{min}}$$

intensity

B_{max}

B_{min}

modulation

modulation

R_1 R_2 R_3

R_1 R_2 R_3

Figure 3   Concept of Modulation

1.0 (normalized)

modulation

cut-off frequency

0

spatial frequency c/mm
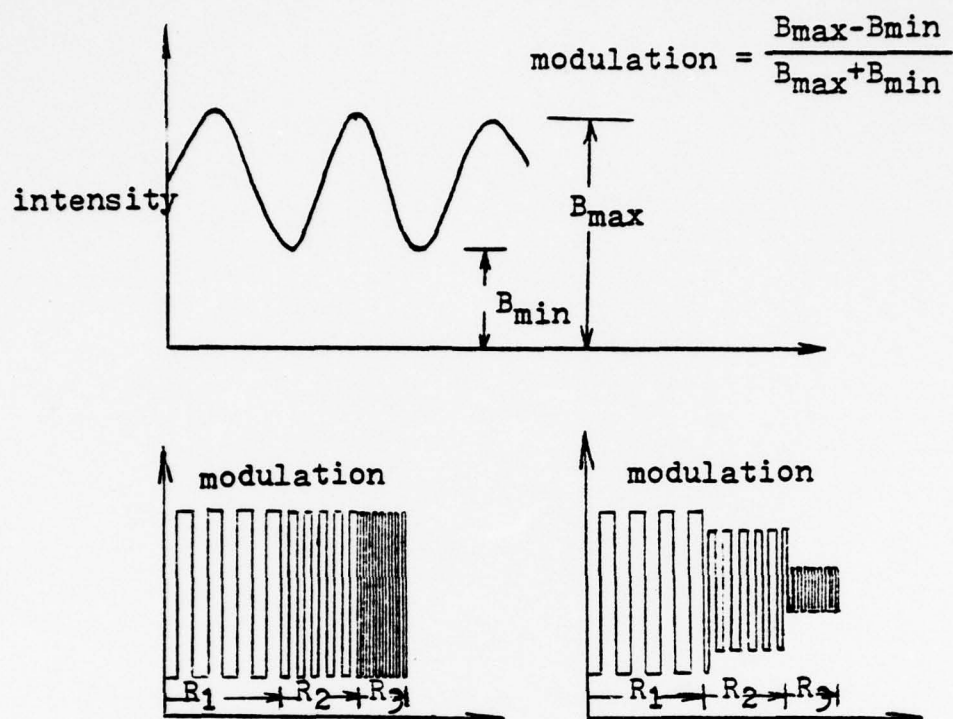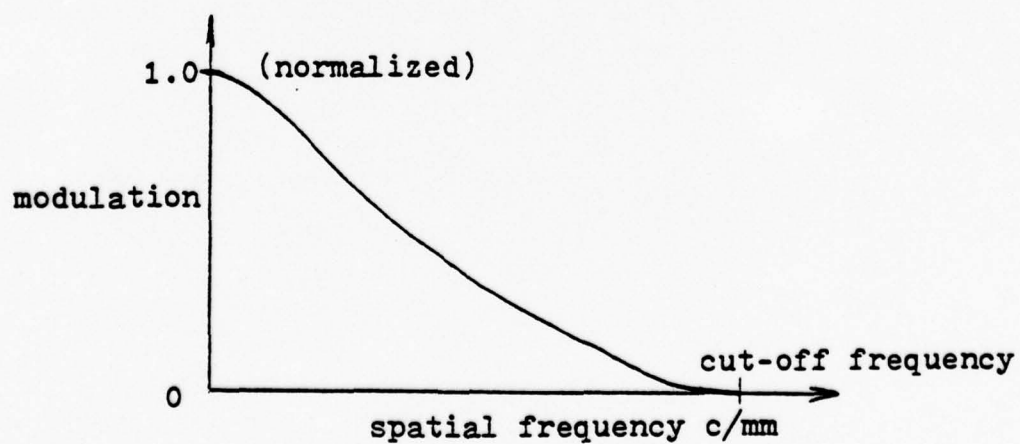
Figure 4 Typical MTF Curve

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OTF of an optical system is likely to depend on the following seven parameters; 1. aperture   2. field angle   3. inclination of test target   4. focal setting   5. color balance of illuminate 6. object distance   7. orientation of lens mount.

One of the oldest mer_t function for image quality is Rayleigh's two-point resolution criterion.   It stated that two point sources of equal intensity are resolvable when they are separated by a distance corresponding to the radius of the first dark ring in the lens diffraction patterns.   Rayleigh's two-point resolution criterion is not all adequate for use as merit function for designing telescopes. In this study, attempt is made to establish tolerable levels of the performance suitable for cassagrain telescopes, the design concept of an optical test system which will allow technical evaluation of the optical system performance , in terms of its MTF, is introduced.

Theory of Image Formation

The theory of image formation can be formulated mathematically by considering the intensity distribution in the image as the sum of contribution of individual points in the object. Thus, if the point spread function (distribution of intensity in the image of a point) for the optical system is known, we can integrate point by point to compute directly the intensity distribution of the image. If $B(u,v)$ and $B'(u',v')$ are the intensity distributions of an incoherently illuminated object and its image , respectively. $G(u'-u,v'-v)$ is the spread function of the optical system, such that

$$B(u',v') = \iint_{-\infty}^{+\infty} B(u,v) \; G(u'-u,v'-v) \; du \; dv \qquad (4)$$

where u,v are reduced coordinates given by

$$u = \frac{2\pi}{\lambda} n \cdot \sin\alpha\xi \qquad , \quad v = \frac{2\pi}{\lambda} n \cdot \sin\alpha\eta \qquad (5)$$
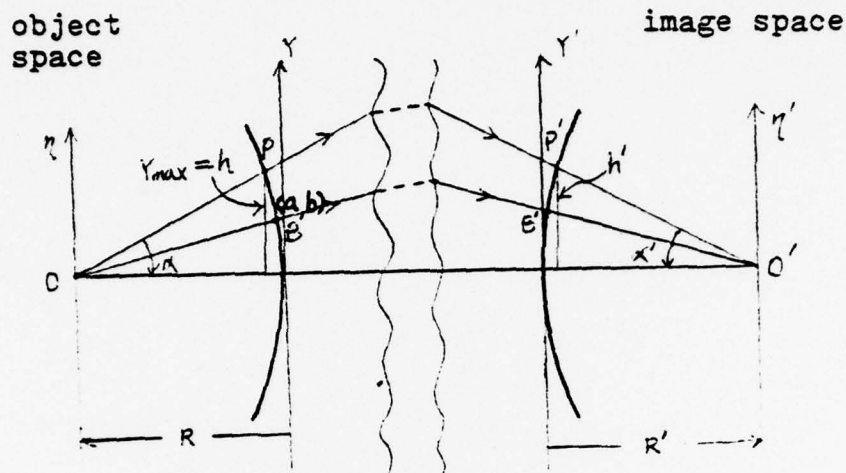
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Figure 5   Canonical Coordinated for the Axial
            Object and Image points

Follow the notation of Hopkins[*] (Figure 5), α being the convergence
angle of the marginal ray with the principal ray, and ξ, η are the
rectangular coordinated of the point in the object plane.   Primes
denote the corresponding quantities in the image space.

Next consider the pupil function $f(x_0, y_0)$, it is defined by

$$f(x_0, y_0) = A(x_0, y_0)e^{ikW(x_0, y_0)} \qquad \text{within aperture} \left.\right\}$$
$$= 0 \qquad\qquad\qquad \text{outside aperture} \left.\right\} \qquad (6)$$

where (x_0, y_0) are reduced coordinates of a point in the pupil.   If
a ray from an object point intersects a reference sphere at the
entrance pupil a point (a, b), and if the radius of the pupil is h,
then

$$x = \frac{a}{h} = \frac{a'}{h'} \qquad \& \qquad y = \frac{b}{h} = \frac{b'}{h'} \qquad (7)$$

If the system is free from aberration, $W(x_0, y_0) = 0$ and $A(x_0, y_0) = 1$,
so that $f(x_0, y_0) = 1$ within the aperture.   With a circular pupil,

therefore, the wave lying outside the circle $x_o^2 + y_o^2 = 1$ will not be transmitted through the system, and $f(x_o, y_o) = 0$ for $(x_o^2 + y_o^2) > 1$. From the result of diffraction theory, the complex amplitude in the image plane can be written as,

$$F(u', v') = \iint_{-\infty}^{+\infty} f(x_o, y_o) \cdot e^{i2(u'x_o + v'y_o)} dx_o dy_o \qquad (8)$$

The intensity in the image plane, $G(u', v')$, is then given by

$$G(u', v') = \left| F(u', v') \right|^2 \qquad (9)$$

The Fourier inverse transform of $G(u', v')$ is represented by $g(s_o, t_o)$, namely point spread function. Then, the application of Parseval's theorem to the above relations gives

$$g(s_o, t_o) = \iint_{-\infty}^{+\infty} f(x_o, y_o) \cdot f^*(x_o - s_o, y_o - t_o) \, dx_o dy_o \qquad (10)$$

which, by a shift of the origin, reduces to

$$g(s_o, t_o) = \iint_{-\infty}^{+\infty} f(x_o + \tfrac{1}{2}s_o, y_o + \tfrac{1}{2}t_o) \cdot f^*(x_o - \tfrac{1}{2}s_o, y_o - \tfrac{1}{2}t_o) \, dx_o dy_o \qquad (11)$$

If $b'(s_o, t_o)$ and $b(s_o, t_o)$ denote the Fourier inverse transforms of $B'(u', v')$ and $B(u, v)$, respectively. Application of the convolution theorem to the equation (4) above gives

$$b'(s_o, t_o) = b(s_o, t_o) \cdot g(s_o, t_o) \qquad (12)$$

The foregoing can, therefore, be summarized simply by stating that, with incoherent illumination, an object intensity function $B(u, v)$ can be analyzed into a Fourier spectrum of spatial frequency $b(s_o, t_o)$. The effect of the lens system is then to modulate each of these Fourier components in both amplitude and phase, since, in general, the factor $g(s_o, t_o)$ in the above relation is complex.

Each Fourier component $b(s_0, t_0)$ appears in the image multiplied by the response $g(s_0, t_0)$ of the optical system. The image intensity distribution $B'(u', v')$ is then a synthesis of these modulated spatial frequencies.

The frequency variables (s_0, t_0) used above can be simply related to the structure of the object or image. By virtue of the inverse Fourier transform relationship between $B(u, v)$ and $b(s_0, t_0)$, the length of one period, u , of the frequency s_0 is given by $u \cdot s_0 = 2\pi$, so that, recalling the defination of u given in (5),

$$us_0 = \frac{2\pi}{\lambda} n\sin\alpha \xi \cdot s_0 = 2\pi$$

$$s_0 = \frac{\lambda}{n\sin\alpha} R \tag{13}$$

where R is number of lines per unit length in the object. An identical expression with primes gives s_0 in terms of the number of lines per unit length in the image.

For an object with a cosine grating of the form

$$B(u, v) = \gamma + \beta\cos 2\pi(us_0 + vt_0) \tag{14}$$

will always have an image of the same form, with intensity distribution given by

$$B'(u', v') = \gamma + \beta T(s_0, t_0) \cdot \cos\left[2\pi(u's_0 + v't_0) + \theta(s_0, t_0)\right] \tag{15}$$

where $T(s_0, t_0)$ and $\theta(s_0, t_0)$ are related by the formula,

$$T(s_0, t_0)\, e^{i\theta(s_0, t_0)} = D(s_0, t_0) = \frac{g(s_0, t_0)}{g(0,\ 0)}$$

$$= \frac{1}{A}\iint_{-\infty}^{+\infty} e^{ik\left[W(x_0 + \frac{1}{2}s_0, y_0 + \frac{1}{2}t_0) - W(x_0 - \frac{1}{2}s_0, y_0 - \frac{1}{2}t_0)\right]} dx_0\, dy_0 \tag{16}$$

where $T(s_0, t_0)$ is the Modulation Transfer Function, $\theta(s_0, t_0)$ is the

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Phase Transfer Function,(PTF), and $D(s_0,t_0)$ is the Optical Transfer Function (OTF), and A is the normalization factor. Comparing the above equation, it demonstrates the fact that the Optical Transfer Function is the normalized inverse Fourier transform of the optical spread function. Therefore, the above formulation reassures;

1) the image of a sinusoidal type object is itself sinusoidal

2) the worst effect of either diffraction or aberration is to reduce the image contrast, if the aberration is asymmetric then the image will displace sideways by an amount that is corresponding to the phase change.

3) if the system is diffraction limited(free from aberration), then the OTF reduces to

$$D(s_0) = 1/A \cdot \iint_{-\infty}^{+\infty} dx_0 dy_0 \qquad (17)$$

the ideal lens MTF for circular aperture is given by

$$Ti(s_0) = \frac{1}{\pi}(2n - \sin 2n) \qquad (18)$$

where $\cos n = \frac{s_0}{2}$, s_0 is known as the reduced spatial frequency.

II. Calculation of Transfer Function

Many methods have been devised for evaluating the OTF for an optical system. Several methods are delineated below:

i) obtain OTF from equation (4), if the spread function $G(u',v')$ is known

ii) since the determination of the actual spread function is somewhat difficult, therefore the approach from the convolution of pupil function over the aperture technique may be used

iii) calculate the wavefront aberration at various points in the aperture.

In fact, calculation of the Optical Transfer Function is a lengthy effort, even when done on a fast computer. Analytical solutions and numerical algorithm for the computation of the integrals have been devised by many researchers, though their techniques various their results are surprisingly consistent.[1] The value of the ideal lens Modulation Transfer Function, $T_i(s_0,t_0)$ has been calculated by using (18) and is reproduced in Table 1.[2] $T_i(s_0,t_0)$ can be used as a basis for extending the criterion for diffraction limited performance to include the effect of a central obstruction and image motion as well as wavefront errors.

$(s_0,0)$	$T_i(s_0,0)$	$(s_0,0)$	$T_i(s_0,0)$
0.00	100	1.10	33.68
0.10	93.64	1.20	28.48
0.20	87.29	1.30	23.51
0.30	80.97	1.40	18.81
0.40	74.71	1.50	14.43
0.50	68.50	1.60	10.41
0.60	62.38	1.70	6.81
0.70	56.36	1.80	3.74
0.80	50.46	1.90	1.33
0.90	44.70	2.00	0.
1.00	39.10		

Table 1   Diffraction limited MTF for a circular aperture

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a) Effects of wavefront error on the image quality

The optical wavefront is a surface of constant phase for the image-forming light. A point source of light generates spherical wavefronts which correspond to the outward radiating light. In a similar manner, light traveling inward with a spherical wavefront will converge to a point (the diffraction effect for the time being is ignored). Figure 6 shows the wavefronts from a point object in process of imaging waves from one point to the other. The purpose of the lens is simply to convert one spherical wavefront surface into another.

Let us now consider the effect of inserting an irregular element into the light path. The random wavefront effects, due to irregularity such as turbulence or imperfections together with the lens aberration produce wavefront disturbances. The light will no longer converge to a single point, this is shown in Figure 7. The greater the magnitude of the wavefront disturbance the more rapid the fluctuation, or the larger the aberration, or the more scattered the directions of the light propagation become. Therefore, the image "point" appears larger.
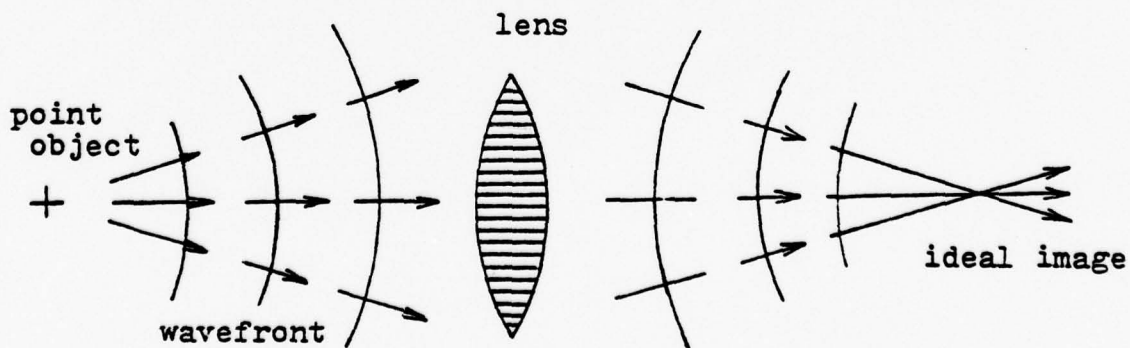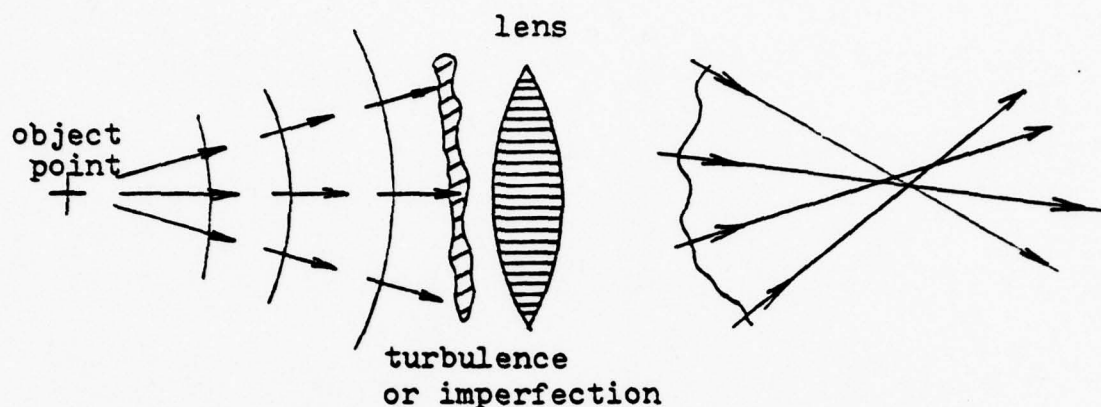
Figure 6 Focusing Effect of Ideal Imaging System

Figure 7 Effect of Optical Wavefront Disturbances

A mathematical description of this effect can be obtained by letting $W(x_o, y_o)$ be the wavefront deformation. The pupil function $f(x_o, y_o)$ may be represented by

$$f(x_o, y_o) = e^{ikW(x_o, y_o)} \qquad \text{when } x_o^2 + y_o^2 \Big\} \, 1$$
$$= 0 \qquad\qquad\qquad x_o^2 + y_o^2 \Big\} \, 1 \qquad (19)$$

Optical quality is often described by the root mean square of the wavefront errors (rms errors). Indeed, various computational and measurement schemes have been developed in the optical industry to determine the rms error and to reduce it as much as possible. Unfortunately, there is no simple relationship between the rms error and an image quality parameter like the optical transfer function (OTF). However, if the rms error is less than $\lambda/8$, the MTF degradation factor may be determined from Figure 8[*3], due to a recent study of Itek. The product of the MTF degradation factor and the aberration-free transfer function yields an approximate transfer function for that wavefront error. The thus calculated

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transfer function is more reliable as the rms error decreases. In general the wavefront is given by the expression

$$\overline{w} = w_{20}\rho^2 + w_{40}\rho^4 + w_{60}\rho^6 + w_{80}\rho^8 + w_{01}y + w_{02}y^2$$
$$+ w_{03}y^3 + (w_{21})^2 y + (w_{22})^2 y^2 + (w_{41})^4 y \qquad (20)$$

where w_{20}, w_{40}, w_{60} and w_{80} represent defocusing 1st, 3rd-, 5th-, and 7th-order spherical aberration, respectively; w_{01} corresponds to a tilt of the aberration function; w_{02} and w_{21} represent primary astigmatism and coma, respectively; w_{22} and w_{41} correspond to 2nd-ary astigmatism and coma; and w_{03} is an ellipitical coma term.
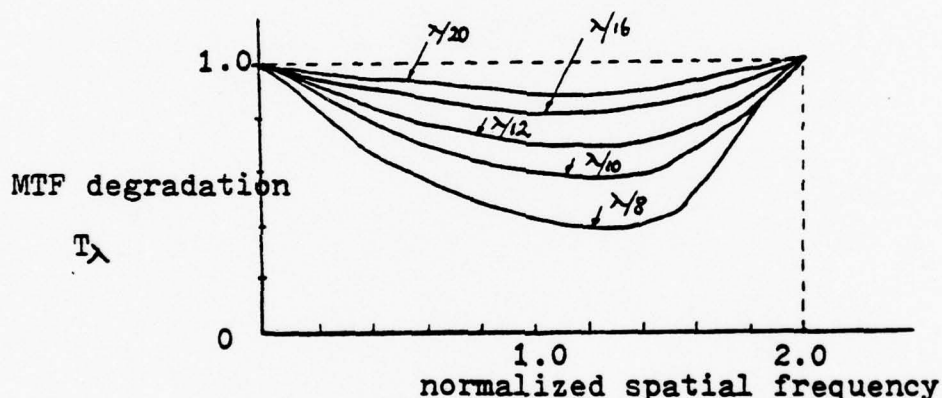

Figure 8 Multiplying factors for various rms wavefront errors

b) Effect of Image Motion

Experiments performed with telescopes usually require longer exposure times. During the course of exposure the image will move. Image motion is always present in any optical system, but it is of small magnitude and does not cause any appreciable error. In general there are four important types of image motion, as shown in Figure 9, due to the work of Rosenau[*4]. Linear image motions arise from uncompensated angular rates, such as inadequate sta-bilization or improper image motion compensation; parobolic image

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motion is due to inherent geometric errors; sinusoidal image motion is due to vibration; and random image motion occurs when many of causes are at work in an unrelated way.
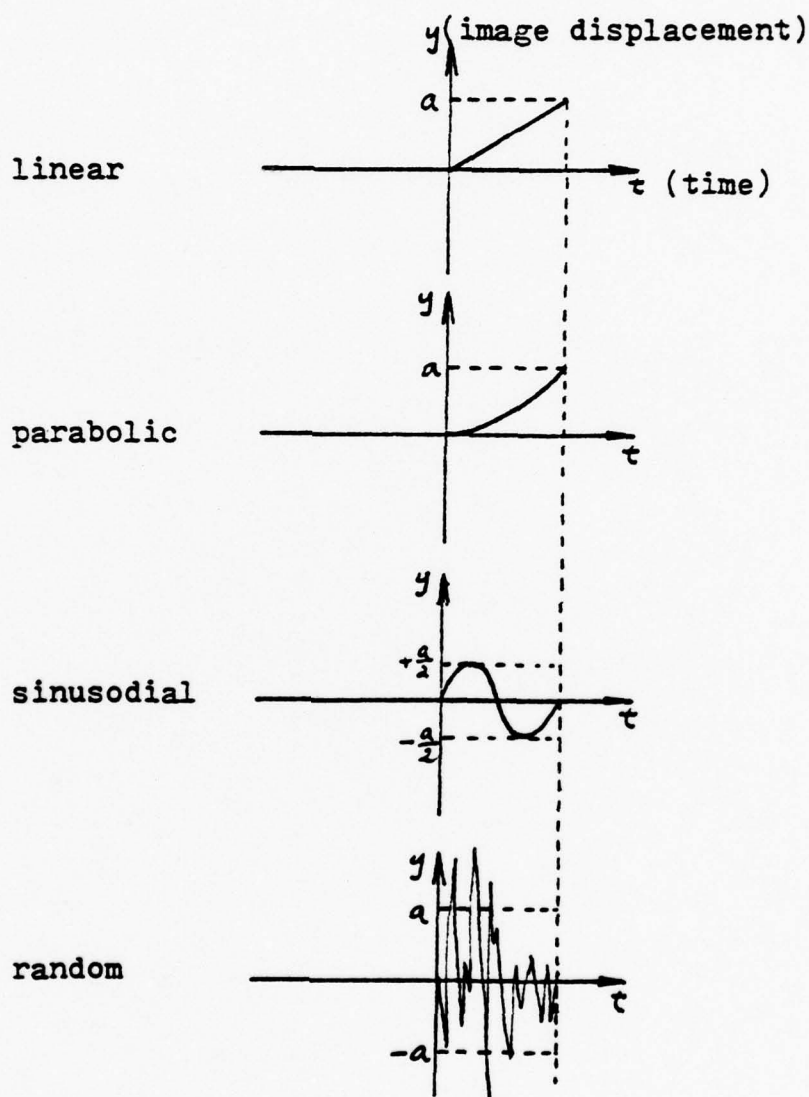

Figure 9 Comparison of Four Important Image Motions

The modulation transfer function due to image motion can be estimated by a graphical technique[5]. The estimate is made by several discrete modulation transfer factors, where each of these

factors are estimated by graphically determining the modulation reduction of an originally fully modulated (MTF = 1.0) sine wave. The modulation fransfer functions of these four types of image motions as given by Rosenau [*4] are shown in Figure 10.   As one numerical example of linear image motion, consider an aerial camera which has exposure time t_e of 1/300 second.   Assume that its focal length f is 6 inches and that the airplane's velocity V is 300 feet/second, and its plane altitude h is 10,000 feet.   In this case, the image velocity at the focal plane, V_i, is

$$V_i = (f/h) \cdot V$$
$$= 0.18 \text{ in/sec.}$$

Thus, the image motion, a, is
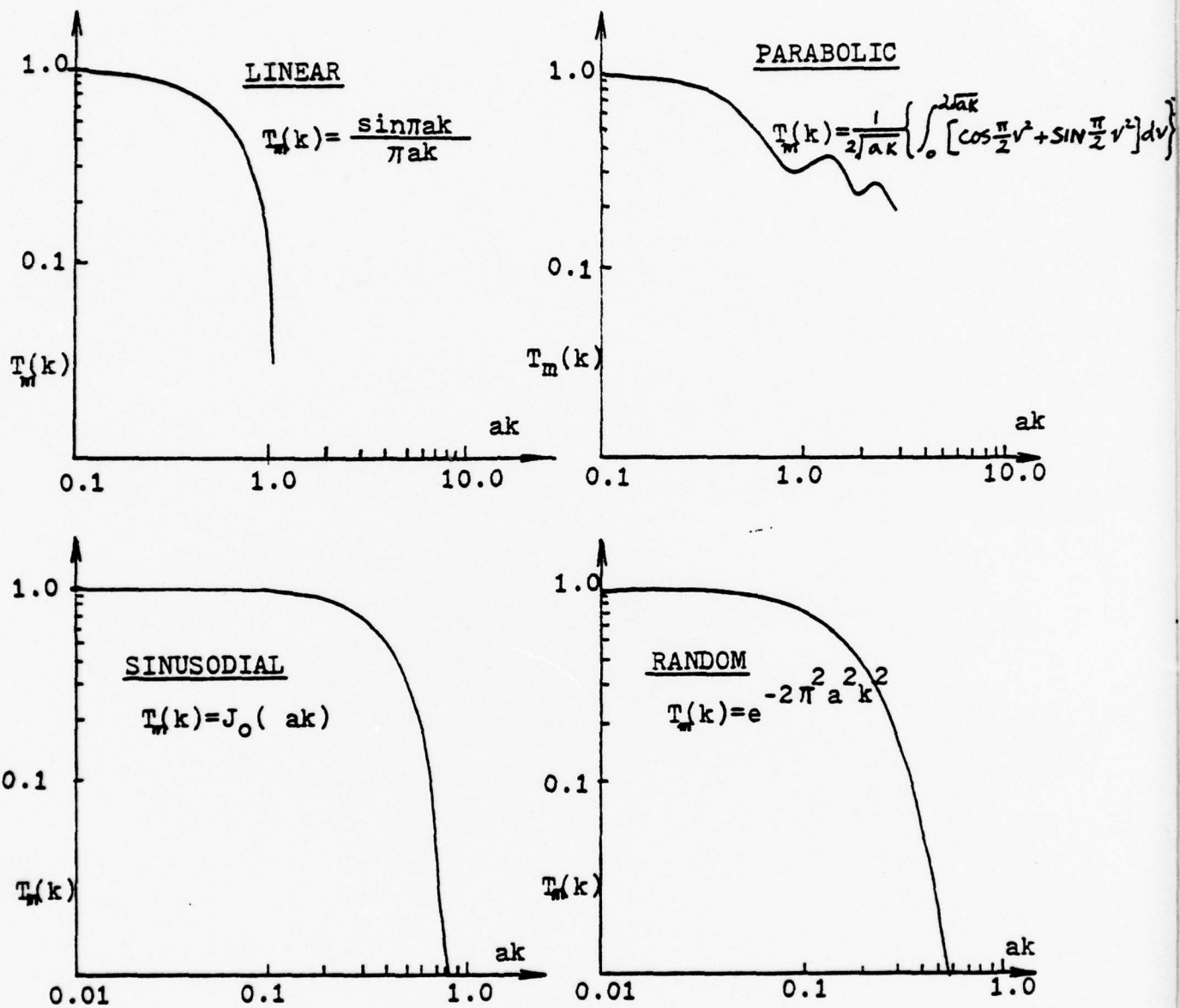
$$a = V_i t_e$$
$$= 0.006 \text{ in} = 1/65 \text{ mm}$$

Consequently, the "cut-off" frequency is 65 cycles/mm (i.e., when a=1/65mm, it is corresponding to k=65 cycles/mm), and the M.T.F. factor at 30 cycles/mm with ak=30/65 ≈ 0.46, is 0.69 according to Figure 10.

The most common image motion encountered in the long exposure for a telescope is the random motion.   The simplest model for this type of image motion is the Gaussian point spread function, the corresponding image motion MTF FACTOR T_m is given by

$$T_m = e^{-2(a\pi k)^2}$$

where a is the magnitude of image motion and k is spatial frequency.

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LINEAR

$$T_m(k) = \frac{\sin \pi ak}{\pi ak}$$

PARABOLIC

$$T_m(k) = \frac{1}{2\sqrt{ak}} \left\{ \int_o^{2\sqrt{ak}} \left[\cos\frac{\pi}{2}v^2 + SIN\frac{\pi}{2}v^2 \right] dv \right\}$$

SINUSODIAL

$$T_m(k) = J_o(ak)$$

RANDOM

$$T_m(k) = e^{-2\pi^2 a^2 k^2}$$

where k=spatial frequency
a=magnitude of image motion
$T_m(k)$ = Modulation Transfer Function
factor due to image motion

Figure 10 Modulation Transfer Functions for the Four Image
Motions Described in Figure 9

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c) Perfect Lens with Central Obstruction

The intensity distribution in space near the geometrical focus
of an error-free pencil of monochromatic light bounded by a
circular aperture has been studied by Zernike and Nijboer, also
by Linfoot and E. Wolf,[6], Asakura and Barakat[7]. The concept of
an annular aperture is particular important in determining the
quality of a cassegrain telescopes. The ratio of obstruction,
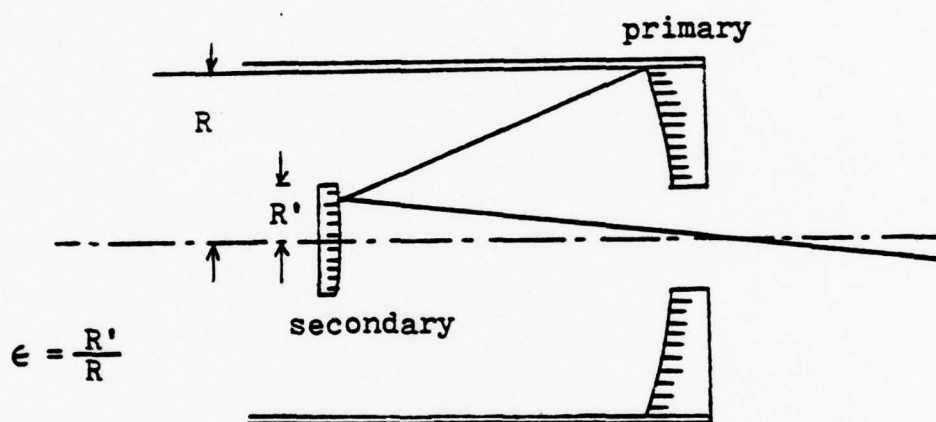ϵ, is defined in Figure 11.

$$\epsilon = \frac{R'}{R}$$

Figure 11   A Typical Cassegrain Telescope

Consider a convergent spherical wave of unit amplitude,
issuing from a circular aperture of radius R and having radius
of curvature $f = \overline{CO}$ (Figure 12), at the moment of emergence.
The complex displacement (amplitude) at a point $P(x,y)$ in the
space near the geometrical focus O is given by[6]

$$U_\lambda^{(R)}(P) = \frac{ikR^2}{f} e^{ik(f-\overline{CP})} \int_0^1 e^{\frac{1}{2}iu\rho^2} J_0(v\rho)\rho d\rho \qquad (21)$$

where
$$u=kR^2z/f^2 \ , \quad v =kRr/f \ , \quad k=2\pi/\lambda \ , \quad r=+(x^2+y^2)^{\frac{1}{2}} \qquad (22)$$

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To allow for the effect of a central obstruction of a radius $R' = \epsilon R$, we substract from $U^{(R)}(P)$ the complex quantity

$$U_\lambda^{(R')}(P) = \frac{ikR'^2}{f} e^{ik(f-\overline{CP})} \int_0^1 e^{\frac{1}{2}iu'\rho^2} J_0(v'\rho)\rho d\rho \qquad (23)$$

and $\quad u' = kR'z/f^2 , \quad v' = kR'r/f \qquad\qquad\qquad (24)$

thus, the intensity at $P(x,y)$ with central obstruction is then the square modulus of the quantity

$$U_\lambda(P) = U_\lambda^{(R)}(P) - U_\lambda^{(R'')}(P)$$

$$= \frac{ikR^2}{f} e^{ik(f-\overline{CP})} \left[\int_0^1 e^{\frac{1}{2}iu\rho^2} J_0(v\rho)\rho d\rho \right.$$

$$\left. - \epsilon^2 \int_0^1 e^{\frac{1}{2}u'\rho^2} J_0(v'\rho)\rho d\rho \right]_{(25)}$$

The problem now is the evaluation of the integrals occuring in equation (25). The standard procedure for its evaluation is the expansion of the exponential in a power series. Table 2 [*8] lists
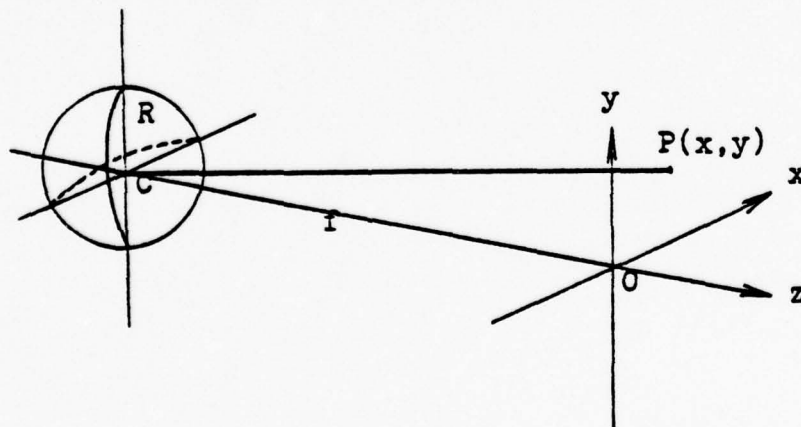

Figure 12   Image Formation of A Circular Aperture

values for the MTF degradation factor, T_ϵ, for central obstruction
for diameter ratios ranging from $\epsilon=0.05$ to $\epsilon=0.60$. When the nor-
malized spatial frequency

$$s/s_o \geqslant \frac{1+\epsilon}{2}$$

the MTF degradation factor will approach to a constant value,

$$T_\epsilon = \frac{1}{1-\epsilon^2}$$

s/s_o	T_i $\diagdown$ ϵ	0.1	0.2	0.3	0.4	0.5	0.6
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
.1	.8729	.988	.966	.941	.907	.859	.788
.2	.7471	.985	.933	.866	.787	.672	.506
.3	.6238	.980	.912	.787	.636	.502	.393
.4	.5046	.974	.881	.763	.626	.490	.383
.5	.3910	.989	.946	.868	.759	.581	.433
.6	.2848		1.040	1.040	.976	.843	.612
.7	.1881				1.183	1.193	1.060
.8	.1041						
.9	.0374	↓	↓	↓	↓	↓	↓
1.0	0	1.003	1.009	1.097	1.183	1.328	1.553
$\frac{1}{1-\epsilon^2}$		1.003	1.010	1.099	1.190	1.333	1.563

*Values for T_i taken from L. Levi,"Applied Optics" , Table 67

Table 2 T_ϵ as a Function of s/s_o

For a perfect lens with central obstruction, the Modulation Tran-
sfer Function is equal to the product of MTF degradation factor (T_ϵ)
and ideal lens MTF (T_i). Figure 13 shows the diffraction limited
modulation transfer function for annuli with various amount of sil-
houetting[*9]. The effect of the annular aperture is thus to decrease
the radius of the first minimum of the diffraction pattern. The
extent to which the minimum is decreased depends upon the amount of

the central region of the aperture.   Unfortunately,this gain in
resolving power is obtained at a considerable loss of intensity in
the diffraction pattern.   A further constraint is the increased
intensity of the secondary maximum as the central obstruction is
increased.   The introduction of a central obstruction reduces the
response at lower frequency and raises it at higher frequencies.
The cut-off frequency remains unchanged.   A system of this type
has a natural tendency to exhibit reduced contrast on course target
and a slightly improved contrast for higher frequencies, particular-
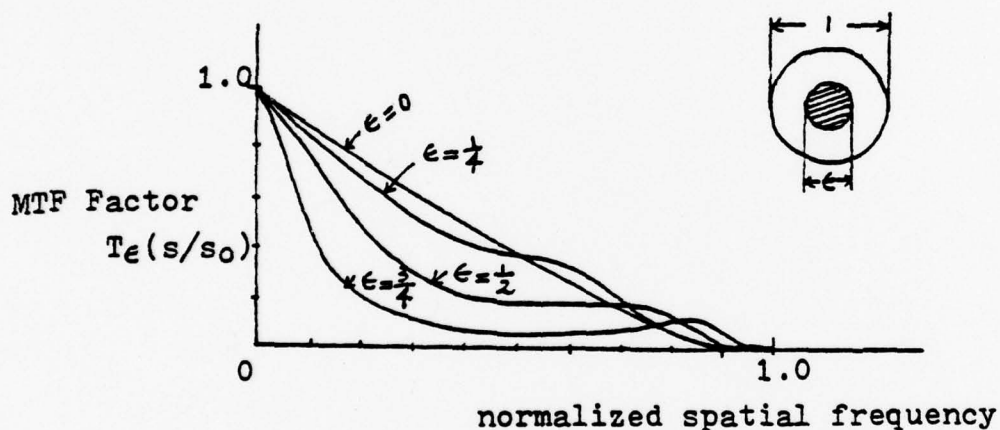ly when the obstruction is large.

Figure 13   Modulation Functions of Annuli

Combination of Wavefront Error, Image Motion and Central Obstruction

The MTF degradation functions for wavefront error, image motion
and central obstruction, T_λ, T_m, and T_ϵ, respectively, are defined
in the previous sections.   Multiplying the ideal lens MTF, T_i, by
any MTF degradation function will give the MTF for a perfect lens
with that particular image degradation under consideration.   These
three degradation functions are independent, so that multiplying the

ideal MTF, T_i, by all three MTF degradation functions will give the complete lens MTF, T,

$$T = T_\lambda \times T_m \times T_\epsilon \times T_i \tag{26}$$

Values for T_λ may be obtained from Figure 8, T_m from Figure 10, and T_ϵ from Table 2, while T_i is given in Table 1.

III. Measuring Instruments and Techniques

The optical transfer function (OTF) or the modulation transfer function (MTF), its modulus, is now generally accepted as one of the suitable objective method of assessing or specifying the image quality of an optical system.   For a full description of the performance of an optical system, however, a large number of OTF or MTF curves are required covering a range of spatial frequency, test azimuths and field positions.   To obtain all this data and then use it to determine whether the optical system has or has not an acceptable performance is very time consuming and expensive. However, the test criteria may be carefully chosen so that to have measurements which are representative of the overall subjective performance of the optical system.

The Theory of Measuring Techniques

Many methods have been utilized over the past decade in order to successfully measure the modulation transfer function of an optical system.   The type of method used is somewhat dependent upon the reason for testing.   Broadly speaking, the techniques can be broken down into two distinct categories: those involving scanning devices and the interferometric type measurements.

A) Scanning Techniques

Consider a narrow incoherently illuminated slit G(u,v) which is being used as a test object for an optical system. The image of the slit (line spread function) has an intensity distribution which can be represented by G(u',v'), as Figure 14.
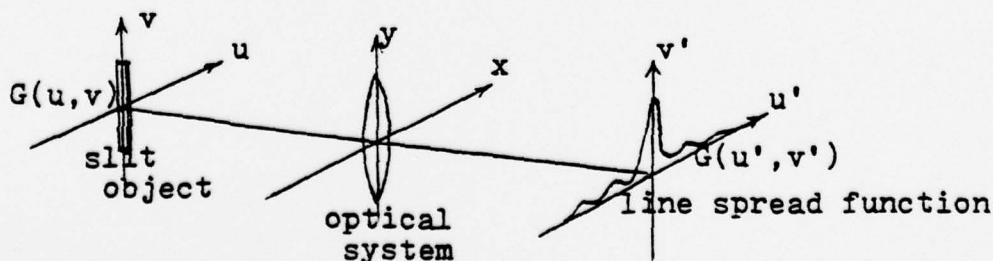
Figure 14   Image of a Slit

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It has previously been shown that the Fourier Transform of
the line spread function is equivalent to the optical transfer
function of the system under testing. Suppose the spread func-
tion falls on a screen whose transmission varies in one direction
and is constant in an orthogonal direction. Thus the optical
transfer function $D(s)$ is given by

$$D(s) = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v')\, e^{-iu's}\, du'dv' \qquad (27)$$

where $\frac{1}{A}$ is photometric constant, and $s=2\pi R''f''$ is the reduced
spatial frequency. Expanding the above equation we have

$$D(s) = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v')\left[\cos(u's) - i\sin(u's)\right] du'dv'$$

$$= \frac{1}{A}\iint_{-\infty}^{\infty} G(u',v')\cos(u's)\, du'dv'$$

$$\qquad - \frac{i}{A}\iint_{-\infty}^{\infty} G(u',v')\sin(u's)\, du'dv'$$

$$= Re - iIm$$

$$= T(s)\, e^{i\theta(s)} \qquad (28)$$

where

$$Re = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v')\cos(u's)\, du'dv' \qquad (29)$$

$$Im = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v')\sin(u's)\, du'dv' \qquad (30)$$

and

$$T(s) = (Re^2 + Im^2)^{\frac{1}{2}} = \text{Modulation Transfer Function} \qquad (31)$$

$$\theta(s) = \tan^{-1}(Im/Re) = \text{Phase Transfer Function} \qquad (32)$$

By far the most critical component in measuring is the target ,
it is not difficult to obtain an MTF if a good sinusoidal grating
is available.   Recognizing the difficulty of producing accurate
sine wave targets, many other forms of target have been used to
improve the results, such as sine-cosine pairs, square wave and
Morie'fringe, as shown in Figure 15, and other types of targets.
A typical MTF measurement for different types of target , Figure16,
thus can be arranged with the help of a specific grating.
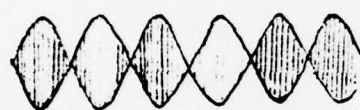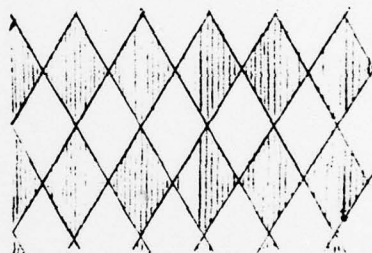

(A) sinusoidal

(C) square wave
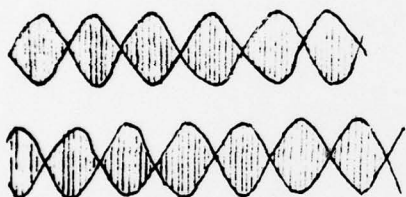
(B) sine-cosine pairs

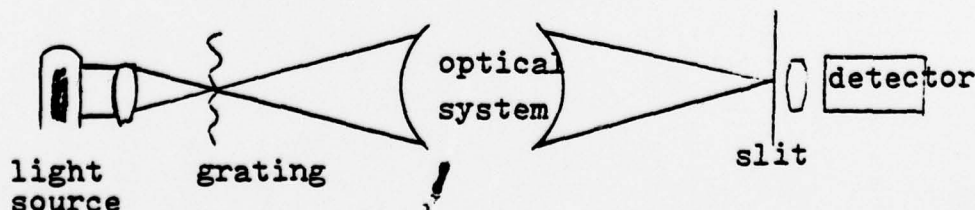(D) Moire fringe

Figure 15   Typical Scanning Screens

Figure 16   Arrangement for MTF measurement

(1) Sinusoidal Screen

If the form of a sinusoidal screen can be represented by an expression of $(1+\sin(u's+\delta))$ and if L is the total amount of light transmitted by the screen then

$$L = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v') \left[1+\sin(u's+\delta) \right] du'dv'$$

$$= \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v') \, du'dv' \; + \; \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v')\sin u's\cos\delta du'dv'$$

$$+ \; \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v') \cos u's \, \sin\delta \, du'dv'$$

$$= K + Im \cos\delta + Re \sin\delta \tag{33}$$

where
$$K = \frac{1}{A}\iint_{-\infty}^{+\infty} G(u',v') \, du'dv'$$

= total light in image which is constant

$$L = K + (Re^2 + Im^2)^{\frac{1}{2}} \sin(\delta+\theta) \tag{34}$$

If δ is varied linearly, L will vary sinusoidally with an amplitude $(Re^2 + Im^2)^{\frac{1}{2}}$. Thus the Modulation Transfer Function can be measured by using an incoherently illuminated slit, the image of which is the spread function denoted by $G(u',v')$ and scanning the image with a series of sinusoidally transmitted targets.

(2) Sine-cosine Pairs Screen

As in the previous method, if a screen of form $\left[1+\sin(u's+\delta) \right]$ is used first. The transmission of the spread function through the target is given by,

$$L = K \, IM \cos\delta + Re \sin\delta$$

$$L - K = Im \cos\delta + Re \sin\delta = L'$$

If the spread function is imaged through a target displaced by $\frac{1}{4}$ cycle (ie. $\delta_2 = \delta_1 + \pi/2$) and if the transmission is now represented by L'_2, it follows that

$$L'_1 = \text{Im} \cos \delta_1 + \text{Re} \sin \delta_1$$

$$L'_2 = \text{Im} \cos \delta_2 + \text{Re} \sin \delta_2$$

$$T(s) = (L_1'^2 + L_2'^2)^{\frac{1}{2}} = (\text{Im}^2 + \text{Re}^2)^{\frac{1}{2}} \tag{35}$$

$$\theta(s) + \delta = \tan^{-1}(L'_1 / L'_2) \tag{36}$$

The significance of equations (34) and (35) is that by merely measuring the transmission through two targets of the same saptial frequency, but changed in phase by $\frac{1}{4}$ cycle, then not only can we obtain an expression for the MTF, but also the phase information is readily extractable.

(3) Square Wave Techniques

If now a square wave target is used, the corresponding response is known as the square wave MTF and is somewhat different from the sine wave response. Assume a rectangular type target being used as a test object, the object $O(x')$ can be represented by a Fourier series,

$$O(x') = B\left\{1 + 4/\pi\left[\cos 2\pi s x' - 1/3\cos 2\pi(3s)x' + 1/5\cos 2\pi(5s)x' + \cdots\cdots\right]\right\}$$

the corresponding image is represented by

$$I(x') = B\left\{1 + 4/\pi\left[T(s)\cos 2\pi s x' - 1/3 T(s)\cos 2\pi(3s)x' + 1/5 T(s)\cos 2\pi(5s)x' + \cdots\cdots\right]\right\}$$

If higher order harmonics in the rectangular wave could be eliminated in some way, the measurement would be equivalent to that obtained by using a sinusoidal type grating.

(4) Morie'Fringe Technique

When two gratings are placed upon one another, they produce a Morie' fringe, the waveform of which is triangular. The Morie' pattern can be used as a test pattern in the measurement of MTF instead of a sinusoidal grating.

(5) Spread Function Technique

In this method of measurement the intensity distribution in the image of a narrow, incoherently illuminated slit source is measured by scanning with a second narrow slit. Then a computer program is utilized to perform a Fourier Transform operation on this distribution to give the optical transfer function. Mathematically,

$$D(s) = \iint_{-\infty}^{+\infty} G(u',v') \, e^{-2isu'\pi} \, du',dv'$$

Thus, if $G(u',v')$ can be measured, then $D(s)$ can thus be computed directly. In actual practice the integral calculation is not difficult to perform by means of a digital computer. A major drawback with this type of measurement is the fact that in order to measure the rapid variations in intensity at the edge of the spread function, a narrow slit is required. Associated with this is a subsequent reduction in the illumination falling onto phototube, thus making an accurate recording of the spread function an extremely difficult process.

(6) Edge Gradient Technique

The optical transfer function can be computed indirectly by measuring the intensity distribution in the image of an edge. If the energy distribution in the image of an edge $E(x)$ can be

measured as a function of x, the distance across the image, then the line spread function can be shown mathematically to be equivalent to the derivative of the edge gradient dE/dx. Since the optical transfer function is the inverse Fourier transform of the line spread function, then it is possible to calculate the optical transfer function from the measurement of the edge function.

To perform this experimentally, the image of the edge must be detected by a narrow slit and a photocell, then the image can be recorded on film and analyzed by means of a microdensitometer. This technique has the advantage that the sinusoidal grating is not necessary; however, the analysis of the intensity distribution in the shadow region is extremely dependent on the signal to noise ratio and measurement made are of doubtful accuracy.

B) Interferometric Techniques

It was first shown by H. H. Hopkins[*10] that the optical transfer function can be directly measured by means of a shearing interferometer. With the advent of lasers in the past ten years, and the need for very high-quality optics both in the laboratory and in field environments, the interferometric techniques appears to be ideally suited to single-wavelength measurements. There are two basic type of interferometric measurements.

(1) Auto-correlation Method

The optical transfer function is previously shown to be equivalent to the autocorrelation of the pupil function f(x,y),

$$D(s) = \frac{1}{A} \iint_{-\infty}^{+\infty} f\left(x+\frac{s}{2},y\right) f^*\left(x-\frac{s}{2},y\right) dxdy \tag{37}$$

where s is the amount of shear of the wavefront and is actually equal to the reduced spatial frequency. The region of integration is the area of overlapped apertures as displayed in Fig. 16. To perform this integration experimentally a two beam shearing interferometer can be used. If the wavefront aberration is represented by W(x,y), then

$$f(x,y) = e^{ikW(x,y)}$$

Thus, if the wavefront aberration can be determined experimentally, the above equation can be applied to calculate the optical transfer function.   Various types of interferometer have been devised for this purpose.
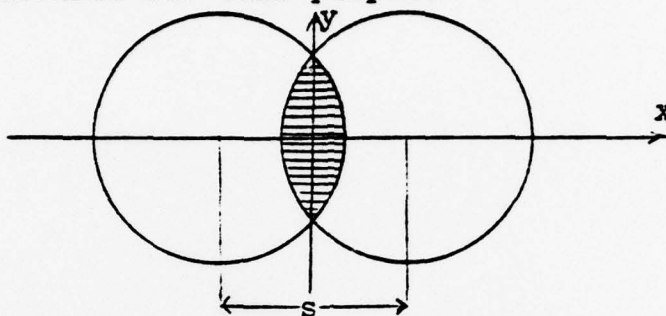

Figure 17   Region of Integration for Frequency s/2

(2) Cross-correlation Method

The basis of the cross-correlation technique is analogus to the random noise, which is often used to measure the frequency response of a communication system.   In this technique there are two developed photographic plates being utilized as random charts ; one is placed in the object plane and the other acts as a scanning screen in the image plane.   The light transmitted through the second plate is measured photoelectrically against the shear and this gives the cross-correlation function.   The Fourier transform of this function is then derived in order to calculate the MTF.

In this method the spatial frequency range is limited for the photographic are not random in higher spatial frequencies and the fact it is not possible to measure the phase makes this technique somewhat unattractive.

From the analysis of the above methods, it is important to consider the degree of difficulty and the accuracy of result of a method before it is chosen to measure the MTF of an optical

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system.   The   trend in the early days of instrumentation
was to introduce new and frequently more complex techniques, and
usually involving sophisticated electronics.   The emphasis has
now been changed as to concentrate more efforts on the calibra-
tion and qualification of the instruments under all expected
conditions of use.   It is also very helpful to consider the test
criterion philosophy on which the design of the instruments is
based.   They may be summarized as these;

* A measurement of the system MTF alone is sufficient (i.e.,
  PTF does not need to be measured)
* Measurement of MTF at low spatial frequencies are most
  significiant
* Measurement of on axis MTF are more important

It is generally accepted that the low spatial frequency region
of the OTF curves is the most significant as far as the subjective
impression of the image quality is concerned, since in general it
has the greatest effect on the contrast of the image and sharpness
of the edge.   Furthermore, at low spatial frequencies the PTF of
the system is usually small and therefore only the system MTF need
to be considered.   The results of measurement also showed that
the low and intermediate spatial frequencies region of the MTF
curve can be characterized with sufficient accuracy by the MTF
at a single spatial frequency.   A good correlation existed between
this value of MTF and the results of subjective tests of image
quality.

In the majority of military sights the performance at the center
of the field is always expected to have the greatest effect on a
subjective evaluation of performance, since the eye will in general
look along the axis of a sight and the user will point the sight in
to bring potential targets into the center of field of view.   This
does not, of course, imply that the performance of the off-axis has
no significance.   In fact for all the sights tested, the off-axis
performance was fairly closely related to the on-axis performance.

Measuring Instruments

The system described here is an extremely flexible one which is designed for its simplicity and can be easily adapted to any special measurement.   At heart of the system is a large granite surface table, flat to better than 0.001" over the entire surface. It serves as an excellent reference from which to set up and align any optical system configuration.   The system may be incorporated into either a single- or a double-pass mode corresponding to finite or infinite conjugate.   Typical arrangements of finite and infinite conjugate tests are shown in Figure 18.   For measuring the system MTF,stable lens holders are required that permit precisely defined adjustments and displacements, thus allowing measurements to be made at known field positions and in desired image planes.

In effect the instruments may be thought of as comprising several modules, as outlined in Figure 19.   The object generator contains the light source and acts as the test object for the system under evaluation.   In general,a He-Ne laser with wavelength equal to 6329.914 Angstrom is used for interferometric methods, while sinusoidal targets are comprised of a tungsten ribbon filament lamp used to incoherently illuminate with a film loop attached to a
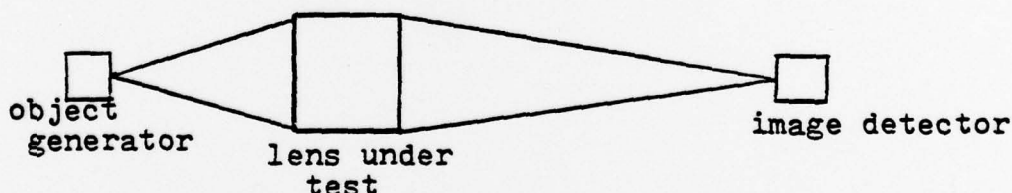

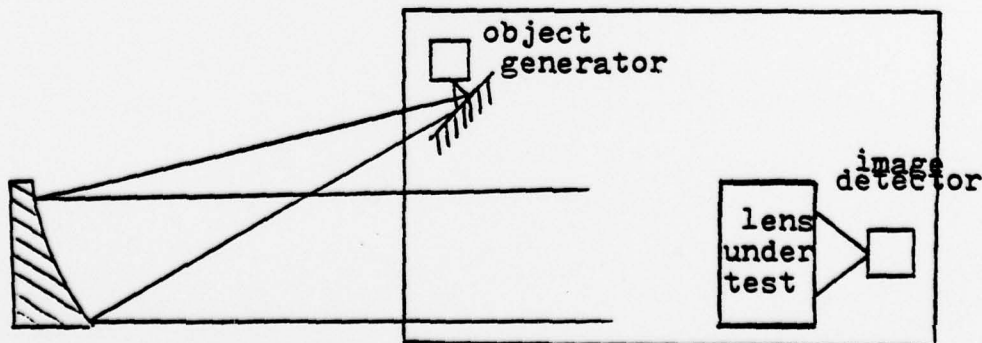Figure 18a   Typical finite conjugate tests

Figure 18b   Typical infinite conjugate tests

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rotatable drum.   The image detector consists of a narrow ruled slit,
suitable collecting optics, and a photomultiplier tube.   Light pass-
ing through the slit is collected by means of a high-aperture lens
and transferred to the photomultiplier tube.   The signal from the
photomultiplier is then passed to a switching arrangement, enabling
the operator to select a method of presentation.

Error Analysis

Errors can originate from many sources, even with a simple system.
Precautions must be taken when MTF measurements are being made. A list
of the principle sources of error in MTF evaluations are listed below:

Environmental errors:

Vibration

Air turbulence

Dust

Temperature and humidity variation

Bench and Collimator errors:

Straightness and flatness of benches

Alignment of benches and collimators

Aberration present in collimator lenses

Object Generator

Partially coherent light in illumination system

Stability of light source

Color temperature of light source

Target errors, modulation, transmission, etc.

Aberration in relay optics

Stray light

Failure to fill aperture of system under test

Image Detector:

Parallelism of slit

Variation of sensitivity of photomultiplier tube face

Response of electronics to range of spatial and temperature
    frequencies

With careful design and experimentation it will be observed that
most , if not all, of the above errors can be eliminated from the
system.

Conclusion

The concepts and techniques outlined here are applicable to a wide range of optical quality assessment needs. Because of the limited time available for this task and the shortage of suitable and more sensitive equipment, the measurements and data analyses are not completed. The author sincerely wishs that an extension will be granted in order to complete the present study.

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*7 Annular & Annulus Aperture with Spherical Aberration and Defocussing, T. Asakura and R. Barakat, Applied Physics of Japan, Vol. 30, No.10, p.728, 1961.

*8 Applied Optics, L. Levi, Table 67.

*9 The Evoluation of the Transfer Function, F. Abbott, Optical Spectra, p.58, March 1970.

*10 H. H. Hopkins, Opt. Acta $\underline{2}$, 23,(1955).